A New Approach for the Symbolic Model Checking of Petri Nets

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June 23, 2020

Ariane 5, 1996

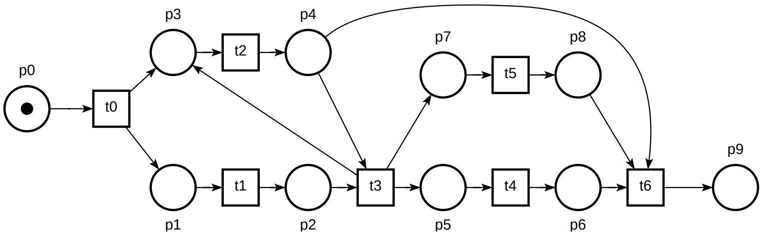
"It is fair to state, than in the digital era correct systems for information processing are more valuable than gold."

— H. Barendregt, The quest for correctness.

seL4, CompCert,
Protocole de cohérence de cache "Futurebus+",
Algorithmes distribués randomisés.

— H. Garavel, Three Decades of Success Stories in Formal Methods.

Mathematical model: Petri net
Introduction



A Petri net example; Christian Stahl.

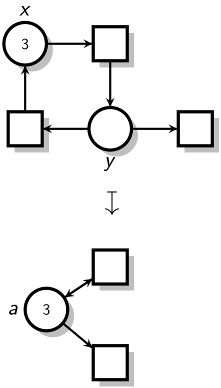
"Decomposing Petri net state spaces." In 18th German Workshop on Algorithms and Tools for Petri Nets. 2011.

- **Context**: Model Checking of "General" Petri nets
  - Not only 1-safe nets
  - Inhibitor and Read arcs
- **Goal**: Use of net reductions to overcome *state-space explosion*
  - Great results for model counting [Berthomieu, 2019]
  - SMT-based methods

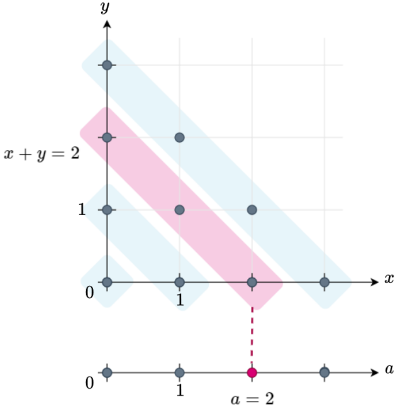
- A property P is correct if for all reachable marking m in $R_N(m_0)$, m satisfies P, denoted $m \models P$
  - proving P correct is equivalent to checking $\Box P$ in LTL or $\mathrm{AG}\, P$ in CTL
- Formula with variables in $\vec{x}$ that is only "satisfiable at marking m": $\underline{m}(\vec{x}) \equiv \bigwedge_{i \in 1..n}(x_i = m(p_i))$
- Check satisfiability of $\neg P(\vec{x}) \wedge \underline{m}(\vec{x})$

Some Examples of Interesting Properties

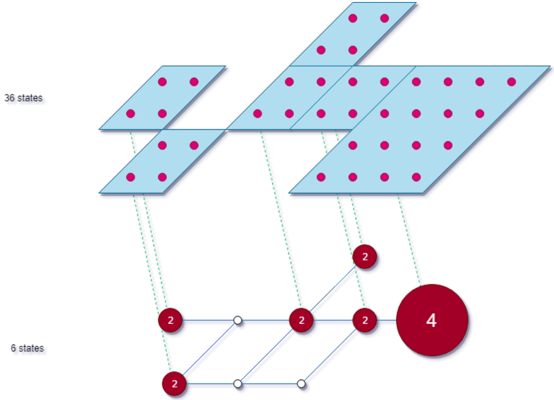
- **PlaceReach**: $\mathrm{REACH}(p) \;\equiv\; m(p) \geq 1$
- **QuasiLiveness**: $\mathrm{LIVE}(t) \;\equiv\; \bigwedge_{p \in {}^\bullet t} \mathrm{COVER}(p, \mathbf{pre}(t, p))$
- **ReachabilityDeadlock**: $\mathrm{DEAD} \;\equiv\; \bigwedge_{t \in T} \neg \mathrm{LIVE}(t)$
- **ConcurrentPlaces**: $p_1 \| p_2 \;\equiv\; \mathrm{REACH}(p_1) \wedge \mathrm{REACH}(p_2)$
- **OneSafe, StableMarking,** ...



Net reduction example, with equation $E : a = x + y$



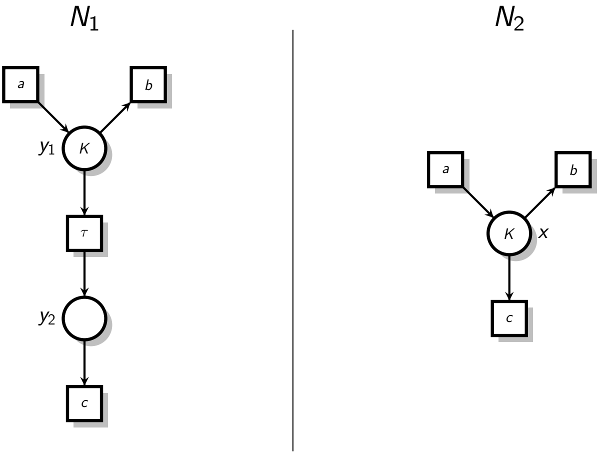
Relation between state-spaces



State-space abstraction by a "polyhedral approach"

Formalization of Net Reductions

Reduction Rule Example: Concatenate (CONCAT)



Equation: $x = y_1 + y_2$

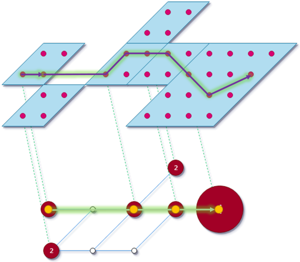
- A marking m can be associated to *system of equations* $\underline{m}(\vec{x})$ defined as, $x_1 = m(p_1), \ldots, x_n = m(p_n)$ where $P = \{p_1, \ldots, p_n\}$
- E is *satisfiable* for m if the system $E, \underline{m}$ has solutions
- Given two markings m_1, m_2 from two nets N_1, N_2, we say that m_1 and m_2 are *compatible*, denoted $(m_1 \uplus m_2)$, when $m_1(p) = m_2(p)$ for all p in $P_1 \cap P_2$ (or equivalently $\underline{m_1}, \underline{m_2}$ is satisfiable)

- E-abstraction: $(N_1, m_1) \sqsupseteq_E (N_2, m_2)$
  - (A1) system E is solvable for N_1, N_2 and the initial markings are compatible with E, meaning $m_1 \uplus m_2 \models E$
  - (A2) for all firing sequence σ_1 such that $(N_1, m_1) \stackrel{\sigma_1}{\Longrightarrow} (N_1, m'_1)$ then for all marking m'_2 over P_2 such that $m'_1 \uplus m'_2 \models E$ we must have a firing sequence σ_2 in N_2 with the same observables, meaning: that $(N_2, m_2) \stackrel{\sigma_2}{\Longrightarrow} (N_2, m'_2)$ and $l_1(\sigma_1) = l_2(\sigma_2)$.

- E-abstraction equivalence: $(N_1, m_1) \rhd_E (N_2, m_2)$
  - Iff $(N_1, m_1) \sqsupseteq_E (N_2, m_2)$ and $(N_2, m_2) \sqsupseteq_E (N_1, m_1)$

- **Bounded Model-Checking**: If $(N_1, m_1) \rhd_E (N_2, m_2)$, then for all marking m'_1 in $R_{N_1}(m_1)$ there exists m'_2 in $R_{N_2}(m_2)$ such that $m'_1 \uplus m'_2 \models E$.

- **Invariance Checking**: If $(N_1, m_2) \rhd_E (N_2, m_2)$, then for all pair of markings m'_1, m'_2 over N_1, N_2 such that $m'_1 \uplus m'_2 \models E$ and $m'_2 \in R_{N_2}(m_2)$ it is the case that $m'_1 \in R_{N_1}(m_1)$.



Axioms: Reduction Rules (CONCAT, etc.)

(COMP) **Composability**

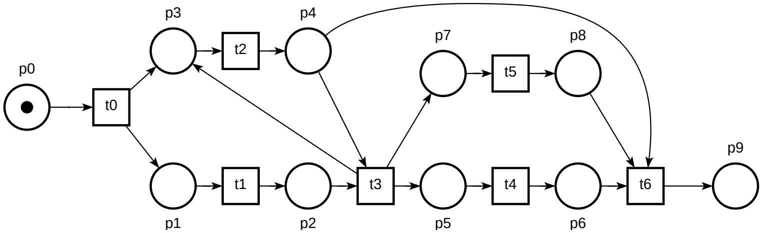
- If $(N_1, m_1) \rhd_E (N_2, m_2)$, then $(N_1, m_1) \| (N_3, m_{,3}) \rhd_E (N_2, m_2) \| (N_3, m_3)$

(TRANS) **Transitivity**

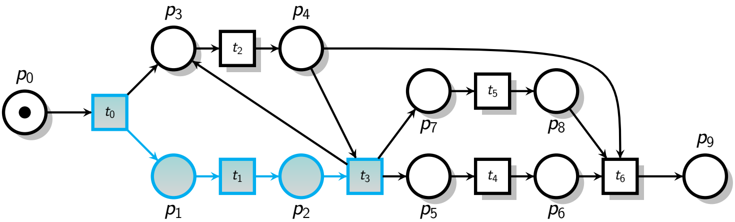
- If $(N_1, m_1) \rhd_E (N_2, m_2)$ and $(N_2, m_2) \rhd_{E'} (N_3, m_3)$, then $(N_1, m_1) \rhd_{E,E'} (N_3, m_3)$.

(RENAME) **Relabeling**

- If $(N_1, m_1) \rhd_E (N_2, m_2)$, then $(N_1[a/b], m_1) \rhd_E (N_2[a/b], m_2)$



Christian Stahl. "*Decomposing Petri net state spaces.*" In 18th German Workshop on Algorithms and Tools for Petri Nets. 2011.

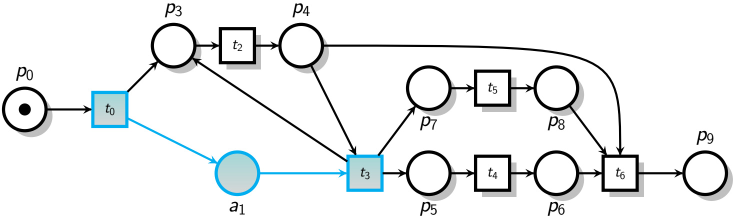


Initial net, S_1, with a pattern for rule (CONCAT) emphasized in blue.

$$E_0 = \emptyset \qquad (1)$$

Net Reduction Example (Step 1)

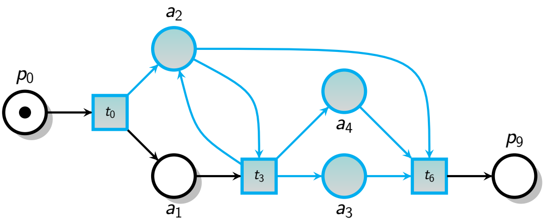
Formalization of Net Reductions



Net S_2, with the result of applying rule (CONCAT) emphasized in blue.

$$E_1 = \left\{ \; a_1 \quad = p_1 + p_2 \right. \qquad (2)$$

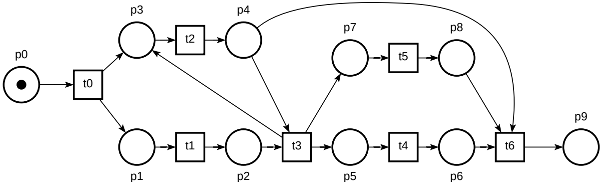
We have: $S_1 \rhd_{E_1} S_2$.



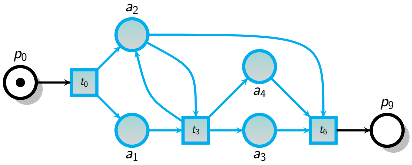
Net S_3, with the result of applying rule (CONCAT) emphasized in blue.

$$E_2 = \begin{cases} a_2 & = p_3 + p_4, \\ a_3 & = p_5 + p_6, \\ a_4 & = p_7 + p_8 \end{cases} \tag{3}$$

We have: $S_2 \rhd_{E_2} S_3$.



Net S_1



Net S_3

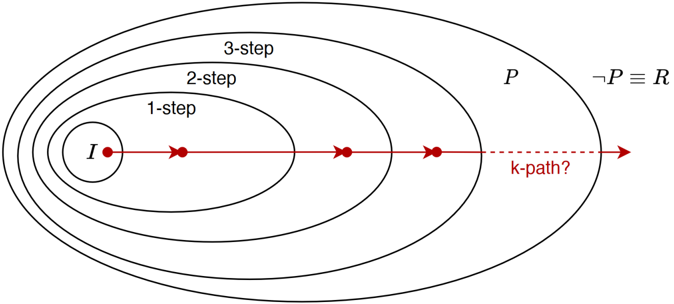
By transitivity, $S_1 \rhd_{E_1,E_2} S_3$

Model Checking Algorithms

- **Bounded Model Checking** (*BMC*): counter-examples
- **Property Directed Reachability** (*IC3*): invariant proof

[Biere et al., 1999]

- Find counter-example violating a property
- Unroll Transitions
- SAT based



BMC method representation

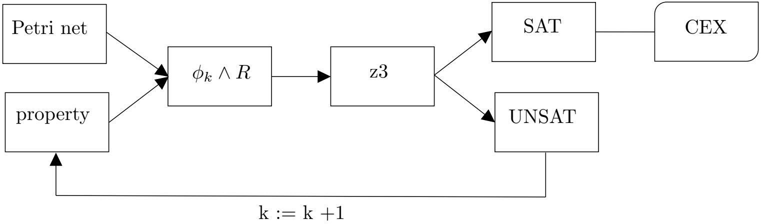
Algorithm adaptation (SMT-based)

- $\mathrm{ENBLD}_t(\vec{x}) \equiv \bigwedge\{(x_i \geq k) \mid k = \mathbf{pre}(t, p_i) > 0\}$
- $\Delta_t(\vec{x}, \vec{x}') \equiv \bigwedge\{(x_i' = x_i + \delta_i) \mid \delta_i = \mathbf{post}(t, p_i) - \mathbf{pre}(t, p_i), 1 \leq i \leq n\}$
- $T(\vec{x}, \vec{x}') \equiv \mathrm{ALLEQ}(\vec{x}, \vec{x}') \vee \bigvee_{t \in T} (\mathrm{ENBLD}_t(\vec{x}) \wedge \Delta_t(\vec{x}, \vec{x}'))$

Lemma: $\underline{m}(\vec{x}) \wedge T(\vec{x}, \vec{x}') \wedge \underline{m}'(\vec{x}')$: m' is at most one-step from m

$$\begin{cases} \phi_0(N, m_0)(\vec{x}_0) & \equiv \underline{m_0}(\vec{x}_0) \\ \phi_{i+1}(N, m_0)(\vec{x}_{i+1}) & \equiv \phi_i(N, m_0)(\vec{x}_i) \land T(\vec{x}_i, \vec{x}_{i+1}) \end{cases}$$

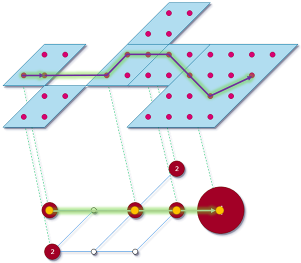
For $k \geq 0$, check $\phi_k(\vec{x}_k) \land \underline{R}(\vec{x}_k)$ until SAT



BMC Algorithm

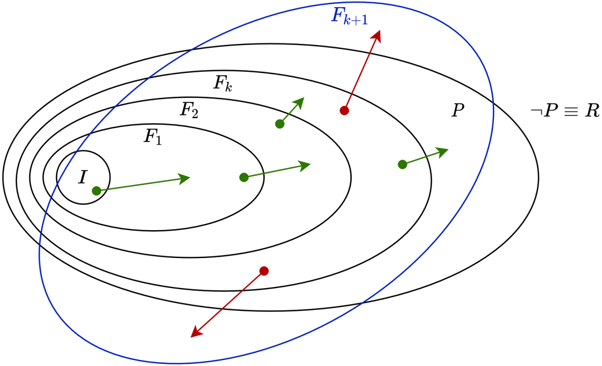
We can find counter-examples to R on N_1 by finding counter-examples to $E \wedge R$ on N_2.
(usually k and $|T|$ are much smaller).

$$\phi_i^r(N_1, m_1)(\vec{x}) \;\equiv\; \phi_i(N_2, m_2)(\vec{y}_i) \wedge E(\vec{x}, \vec{y}_i) \wedge R(\vec{x})$$



[Bradley, 2011]

- Induction, Over-approximation & SAT Solving
- Unroll at most one transition
- Generate clauses that are inductive



IC3 method representation

SMPT: Another Model-Checker

- Available on GitHub under GPLv3 license
  `github.com/nicolasAmat/SMPT`

- Python language ($\approx$ 3,000 LoC)
- Z3 (SMT-LIB v2)
- Input Petri nets at the `.net` format

- Run the tool: `./smpt.py --deadlock <.net>`
- Take advantage of net reductions
  `./smpt.py --deadlock <.net> --reduced <.net>`

Property verification

- Deadlock `--deadlock`
- Quasi-liveness `--liveness <t>`
- (Place) Reachability `--reachability <p>`
- Concurrent Places: `--concurrent-places <p1>,...,<pk>`

Debug

- Verbose: `--verbose`
- Print SMT-LIB input/output `--debug`

We check if a particular place can be marked in the model.

Model	# states	Result	Time	T_{Reduced}
AirplaneLD–10	$4.3\,10^4$	CEX	9.17s	0.16s
AirplaneLD–20	$3.1\,10^5$	CEX	50.26s	0.16s
AirplaneLD–∞	∞	CEX	*n.a.*	0.16s
IBM319 (merge...)	$2.4\,10^3$	CEX	$> 200s$	0.14s
IBM319 (callTo...)	$2.4\,10^3$	PROOF	$> 200s$	12.02s

We check if places $P1$ and $P2$ can be marked together in model AirplaneLD (we know it is not possible)[1].

MODEL	# STATES	RESULT	TIME	T_{Reduced}
AirplaneLD–10	$4.3\,10^4$	PROOF	1.50s	0.26s
AirplaneLD–20	$3.1\,10^5$	PROOF	2.51s	0.26s
AirplaneLD–4000	$2.1\,10^{12}$	PROOF	1 680s	0.26s[2]

[1] time to generate the state space of AirplaneLD-4000 with ITS is $> 2\,500s$.
[2] time to reduce: 67.79s

Application: Concurrent Places Problem

- Useful for the decomposition into Nested-Unit Petri Nets (NUPNs)
- Two places p_1 and p_2 are concurrent, denotes as $p_1 \| p_2$ iff there exists a reachable marking m in $R_N(m_0)$ such that $m(p_1) > 0$ and $m(p_2) > 0$.

A new method that take advantage of net reductions:

(Step 1) Compute the concurrency relation of the reduced net N_2

(Step 2) *Change of Basis*, compute the concurrency relation of the initial net N_1 from the system of equations E and the concurrency relation of the reduced net N_2

- *Concurrency relation*: undirected graph (P, R), where vertices are places and there is an edge $(p, q) \in R$ when $p \| q$

Output: Concurrency relation $\mathcal{C}$
$\mathcal{C} \longleftarrow \{\}$;
$m \longleftarrow$ initial marking m_0;
while $\mathcal{C} \longleftarrow \mathcal{C} \cup$ *stepper*$(m, \mathcal{C})$;
do
  parallel
    begin
      if *IC3 proves that we found all concurrent places*
        then return $\mathcal{C}$;
    begin
      if *BMC finds a counter-example* m' *with new concurrent places* **then**
        $m \longleftarrow m'$;
        continue;

Change of Basis using Reduction Equations

Application: Concurrent Places Problem

```
# R |- P3 = P2
# A |- a1 = Pout1 + Pm1
# A |- a2 = Pback1 + a1
# A |- a3 = Pout2 + Pm2
# A |- a4 = Pback2 + a3
# A |- a5 = Pout3 + Pm3
# A |- a6 = Pback3 + a5
# A |- a7 = Pout4 + Pm4
# A |- a8 = Pback4 + a7
# A |- a9 = a8 + P4
# R |- a9 = 5
# R |- a6 = a4
# A |- a10 = a4 + P2
# R |- a10 = 5
# A |- a11 = a2 + P1
# R |- a11 = 5
```

Output of tool **reduce** on the Kanban instance for $N = 5$ (#states: 2 546 400 – 16 places, 16 transitions, 40 arcs)

Change of Basis using Reduction Equations

Application: Concurrent Places Problem

```
# R |- P3 = P2
# A |- a1 = Pout1 + Pm1
# A |- a2 = Pback1 + a1
# A |- a3 = Pout2 + Pm2
# A |- a4 = Pback2 + a3
# A |- a5 = Pout3 + Pm3
# A |- a6 = Pback3 + a5
# A |- a7 = Pout4 + Pm4
# A |- a8 = Pback4 + a7
# A |- a9 = a8 + P4
# R |- a9 = 5
# R |- a6 = a4
# A |- a10 = a4 + P2
# R |- a10 = 5
# A |- a11 = a2 + P1
# R |- a11 = 5
```

```
# R |- a11 = 5
# A |- a11 = a2 + P1
# A |- a2 = Pback1 + a1
# A |- a1 = Pout1 + Pm1
```

Output of tool **reduce** on the Kanban instance for $N = 5$
(#states: 2,546,400 – 16 places, 16 transitions, 40 arcs)

Conclusion

- The approach used in SMPT is promising
- Contributions for SMT-based model-checking algorithms
- New equivalence relation: *E-abstraction equivalence*
- New method for the *Concurrent Places Problem*

Thank you for your attention!
Any questions?

Bounded Model Checking (BMC)

Init	Step 1	Step 2
$\underline{m_0}(\overrightarrow{x_0})$	$\underline{m_0}(\overrightarrow{x_0})$	$\underline{m_0}(\overrightarrow{x_0})$
$\underline{R}(\overrightarrow{x_0})$	$T(\overrightarrow{x_0}, \overrightarrow{x_1})$	$T(\overrightarrow{x_0}, \overrightarrow{x_1})$
	$\underline{R}(\overrightarrow{x_1})$	$T(\overrightarrow{x_1}, \overrightarrow{x_2})$
		$\underline{R}(\overrightarrow{x_2})$

Assertion stack

Init	Step 1	Step 2
$\underline{R}(\vec{x})$	$\underline{R}(\vec{x})$	$\underline{R}(\vec{x})$
$\underline{m_0}(\overrightarrow{y_0})$	$\underline{m_0}(\overrightarrow{y_0})$	$\underline{m_0}(\overrightarrow{y_0})$
$E(\vec{x}, \overrightarrow{y_0})$	$T(\overrightarrow{y_0}, \overrightarrow{y_1})$	$T(\overrightarrow{y_0}, \overrightarrow{y_1})$
	$E(\vec{x}, \overrightarrow{y_1})$	$T(\overrightarrow{y_1}, \overrightarrow{y_2})$
		$E(\vec{x}, \overrightarrow{y_2})$

Assertion stack with reductions

Over-Approximated Reachability Sequence (OARS) of formulas $F_0, \ldots, F_{k+1}$ such that:

- $(F_0 = I \subseteq F_1 \subseteq \cdots \subseteq F_{k+1} = P)$
- For all $i \in 0 \ldots k+1$. $\underline{F_i}(\vec{x}) \wedge T(\vec{x}, \vec{x}') \Rightarrow \underline{F_{i+1}}(\vec{x}')$

Each F_i describes a set of states that:

1. Includes the states s less than i steps from I,
2. Contains only states s which are more than $k - i + 1$ steps from R.

Proved when $F_i = F_{i+1}$.

Perspectives

- Continue to work on SMT-based algorithms
  - Add states equations
  - Add invariants
  - Add BDDs
- Explore new reduction rules
  - Theorem Prover
  - Specific rules
- Model Counting
  - Convex analysis [Barvinok]
  - Combinatorial approach

Participation in *Reachability* category of the Model Checking Contest.

Prevalence of Reductions over the MCC Instances